Youth Competition Times

UPPSC
LT Grade/GIC/GDC
PHYSICS
Mains
Chapterwise Descriptive Solved Papers

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CONTENT

PHYSICS

01. General Physics & Mechanics

Vector Algebra : Vector and Scalar Quantities, Scalar and Vector Products, Vector Identities

Q. $\vec{L}=1$ **और** $\vec{S}=\frac{1}{2}$ **(दोनों h की इकाई में) के लिए $\vec{L}.\vec{S}$ के सम्भावित मान ज्ञात कीजिए।**

Calculate the possible values of $\vec{L}.\vec{S}$ For $\vec{L}=1$ and $\vec{S}=\frac{1}{2}$ (both in units of h).

UPPCS (Mains) 2003 Paper-II

Ans. : Given,

$\vec{L}=1$ and $\vec{S}=\frac{1}{2}$ the total angular momentum quantum number J,

$$J=\left|\vec{L}-\vec{S}\right|,|\vec{L}+\vec{S}|$$

$$J=\left|1-\frac{1}{2}\right|,\left|1+\frac{1}{2}\right|$$

$$J=\frac{1}{2},\frac{3}{2}$$

For an electron in an atomic system, the spin-orbit interaction is expressed as

$$\vec{L}.\vec{S}=\frac{1}{2}[J(J+1)-L(L+1)-S(S+1]h^2$$

For J = $\frac{3}{2}$,

$$\vec{L}.\vec{S}=\frac{1}{2}\left[\frac{3}{2}\left(\frac{3}{2}+1\right)-1(1+1)-\frac{1}{2}\left(\frac{1}{2}+1\right)\right]h^2$$

$$\vec{L}.\vec{S}=\frac{1}{2}\left[\frac{3}{2}\left(\frac{5}{2}\right)-2-\frac{3}{4}\right]h^2=\frac{1}{2}h^2$$

For J = $\frac{1}{2}$,

$$\vec{L}.\vec{S}=\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}+1\right)-1(1+1)-\frac{1}{2}\left(\frac{1}{2}+1\right)\right]h^2$$

$$\vec{L}.\vec{S}=\frac{1}{2}\left[\frac{1}{2}\left(\frac{3}{2}\right)-2-\frac{3}{4}\right]h^2=-h^2$$

The two possible values of the spin-orbit coupling for L = 1 and S = $\frac{1}{2}$ are $\frac{h^2}{2}$ and h^2

Q. **दर्शाइए कि बलः $\vec{F}=\left(2xy+z^2\right)\hat{i}+x^2\hat{j}+2xz\hat{k}$ एक संरक्षित बल है। इसका स्थिति फलन ज्ञात कीजिए।**

Show that the force $\vec{F}=\left(2xy+z^2\right)\hat{i}+x^2\hat{j}+2xz\hat{k}$ is a conservative force. Find its potential function.

UPPCS (Mains) 2006 Paper-I

Ans. : Force $\vec{F}=(2xy+z^2)\ \hat{i}+x^2\hat{j}+2xz\hat{k}$ is a conservative force, we need to prove that its curl is zero.

Calculate the curl of F

The curl of F is given by,

$$\nabla\times F=\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy+z^2 & x^2 & 2xz \end{vmatrix}$$

Expand the determinant

$$\nabla\times F=i\left[\frac{\partial}{\partial y}(2xz)-\frac{\partial}{\partial z}\left(x^2\right)\right]-J\left[\frac{\partial}{\partial x}(2xz)-\frac{\partial}{\partial z}\left(2xy+z^2\right)\right]$$

$$+k\left[\frac{\partial}{\partial x}\left(x^2\right)-\frac{\partial}{\partial y}\left(2xy+z^2\right)\right]$$

Simplify the curl

$$\nabla\times F=i(0-0)-j(2z-2z)+k(2x-2x)$$

$$\nabla\times F=0\hat{i}+0\hat{j}+0\hat{k}\quad=0$$

Conclusion

Since the curl of F is zero, F is a conservative force.

Find the potential function:

To find the potential function V (x, y, z) such that F = – ∇V, we need to solve the following equations:

$$-\partial v/\partial x\ =2xy+z^2$$

$$-\partial v/\partial y\ =x^2$$

$$-\partial v/\partial z\ =2xz$$

Integrate the first equation

$$V(x,y,z)=-\int\left(2xy+z^2\right)dx$$

$$V(x,y,z)=-x^2y-z^2x+f(y,z)$$

Differentiate V with respect to y and compare with the second equation

$$\partial v/\partial y=-x^2+\partial f/\partial y=-x^2$$

This implies that $\partial f/\partial y=0$. So $f(y,z)=g(z)$

Differentiate V with respect to z and compare with the third equation

$$\partial v/\partial z=-2xz+g'(z)=-2xz$$

This implies that g'(z) = 0, so g(z) = c potential function $V(x, y, z) = -x^2y - z^2x + c$

The potential function is $V(x, y, z) = -x^2y - z^2x + c$

Where, c is a constant.

Q. **A charge of 2 micro-coulomb is moving with velocity $\overline{v}=\left(\hat{i}+\hat{j}\right)\times10^6$ m/sec in a uniform electric field $\overline{E}=\left(2\hat{i}+3\hat{j}-4\hat{k}\right)\times10^6$ volt/m, and magnent field $\overline{B}=\left(\hat{i}+\hat{j}+\hat{k}\right)$ Weber/m^2. Calculate the net force acting on the charge. Here $\hat{i},\hat{j},\hat{k}$ denote, respectively, the unit vectors along the x, y and z axes.**

2 माइक्रो कूलाम का एक आवेश $\vec{v} = (\hat{i}+\hat{j})\times 10^6$ मीटर/से के वेग से एक सम विद्युतीय क्षेत्र $\vec{E} = (2\hat{i}+3\hat{j}-4\hat{k})\times 10^6$ वोल्ट/मीटर तथा चुम्बकीय क्षेत्र $\vec{B} = (\hat{i}+\hat{j}+\hat{k})$ वेबर/मीटर^2 में गति कर रहा है। आवेश पर कार्यरत कुल बल का मान ज्ञात कीजिए। यहाँ $\hat{i},\hat{j},\hat{k}$ क्रमशः x, y, z अक्षों के अनुदिश एकांक सदिश हैं।

UPPCS (Mains) 1996 Paper-II

Ans. : Given, Velocity $\vec{v} = (\hat{i}+\hat{j})\times 10^6\,\text{m/sec}$

Charge q = 2 micro-coulomb = 2×10^{-6}C

Electric field $\vec{E} = (2\hat{i}+3\hat{j}-4\hat{k})\times 10^6\,\text{volt/m}$

Magnet field $\vec{B} = (\hat{i}+\hat{j}+\hat{k})\,\text{Weber/m}^2$

• Calculate the cross-product ($\vec{v}\times\vec{B}$)

$$\vec{v}\times\vec{B} = \begin{vmatrix} i & j & k \\ 10^6 & 10^6 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\vec{v}\times\vec{B} = \hat{i}(10^6\times 1 - 1\times 0) - \hat{j}(10^6\times 1 - 1\times 0) + k(10^6\times 1 - 10^6\times 1)$$

$$\vec{v}\times\vec{B} = (1\hat{i}-1\hat{j}+0\hat{k})\times 10^6\,\text{volt/m}$$

• Calculate net force (F)-

$$\vec{F} = q[\vec{E}+(\vec{v}\times\vec{B})]$$

$$[\vec{E}+(\vec{v}\times\vec{B})] = [(2\hat{i}+3\hat{j}-4\hat{k})\times 10^6 + (1\hat{i}-1\hat{j}+0\hat{k})\times 10^6]$$

$$= (3\hat{i}+2\hat{j}-4\hat{k})\times 10^6\,\text{volt/m}$$

$$\vec{F} = q[\vec{E}+(\vec{v}\times\vec{B})]$$

$$\vec{F} = (2\times 10^{-6})\times[(3\hat{i}+2\hat{j}-4\hat{k})\times 10^6\,\text{volt/m}]$$

$$\vec{F} = 2\times 10^{-6}\times 10^6\times(3\hat{i}+2\hat{j}-4\hat{k})$$

$$\vec{F} = 6\hat{i}+4\hat{j}-8\hat{k}$$

So, Net force acting on the charge-

$$\vec{F} = (6\hat{i}+4\hat{j}-8\hat{k})\text{N}$$

Magnitude of the force is-

$$|\vec{F}| = \sqrt{(6)^2+(4)^2+(-8)^2}$$

$$|\vec{F}| = \sqrt{36+16+64} = \sqrt{116}$$

$$|\vec{F}| \approx 10.77\,\text{N}$$

Background of Vector Calculus, Concept of line, Surface and Volume Integrals, physical meaning of gradient, divergence and curl, Gauss's and Stoke's theorem and Applications

Q. एकसमान आवेशित अर्द्धवृत्तीय चाप के केन्द्र पर विद्युत क्षेत्र ज्ञात कीजिए।

Find the electric field at the center of a uniformly charged semicircular arc.

UPPCS (Mains) 2013 Paper-II

Ans. Consider a semi-circular arc of radius R and total charge Q.

The linear charge density is $(\lambda) = \dfrac{Q}{\pi R}$

Consider an infinitesimal charge element dq = λRdθ at an angle θ from the horizontal axis.

The electric field dE at the center points radially away from dq.

By symmetry, the horizontal components of the electric field from elements on either side cancel out. Only the vertical components (Perpendicular to the diameter) add up.

The vertical component is $dE_y = dE\sin(\theta)$.

The magnitude of dE is given by coulomb's law:

$$dE = \frac{kdq}{R^2} = \frac{k\lambda R d\theta}{R^2} = \frac{k\lambda}{R}\,d\theta$$

Where, $k = \dfrac{1}{4\pi\varepsilon_o}$

Now, the total electric field is the integral of the vertical components from θ = 0 to θ = π :

$$E = \int_0^\pi dE_y = \int_0^\pi dE\sin(\theta) = \int_0^\pi \frac{k\lambda}{R}\sin(\theta)d\theta$$

The constant terms can be taken out of the integral

$$E = \frac{k\lambda}{R}\int_0^\pi \sin\theta\, d\theta$$

The integral of sin (θ) is (–cosθ)

$$E = \frac{k\lambda}{R}[-\cos(\theta)]_0^\pi = \frac{k\lambda}{R}(-\cos(\pi)-(-\cos(0)))$$

$$E = \frac{k\lambda}{R}(-(-1)-(-1)) = \frac{k\lambda}{R}(1+1) = \frac{2k\lambda}{R}$$

Substitute $\lambda = \dfrac{Q}{\pi R}$ and $k = \dfrac{1}{4\pi\varepsilon_o}$ back into the equation

$$E = \frac{2kQ}{\pi R^2} = \frac{2}{R^2}\left[\frac{1}{4\pi\varepsilon_o}\right]\left[\frac{Q}{\pi}\right] = \frac{2Q}{4\pi^2\varepsilon_o R^2} = \frac{Q}{2\pi^2\varepsilon_o R^2}$$

The electric field at the center of a uniformly charged semicircular arc is:

$$E = \frac{2k\lambda}{R} = \frac{Q}{2\pi^2\varepsilon_o R^2}$$

Q. $\nabla^2 V = -\dfrac{e}{\varepsilon_0}$ सम्बन्ध को स्थापित कीजिए। Establish the relation $\nabla^2 V = -\dfrac{e}{\varepsilon_0}$

UPPCS (Mains) 2013 Paper-II

Ans. Gauss's law in electrostatics :-

$$\nabla\cdot E = \frac{e}{\varepsilon_0}$$

Electric field is negative gradient of the electro-static potential :

$$E = -\nabla V$$

Substitute into Gauss's law–

$$\nabla.(-\nabla V) = \frac{e}{\varepsilon_0}$$

$$-\nabla^2 V = \frac{e}{\varepsilon_0}$$

$$\nabla^2 V = -\frac{e}{\varepsilon_0}$$

Q. गॉस नियम का अवकलन रूप प्राप्त कीजिए।

Derive the differential form of Gauss law.

UPPCS (Mains) 2013 Paper-II

Ans. According to Gauss's law, the electric flux through a closed surface is proportional to the enclosed charge Q_{inside}

$$\int_{box} \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_o} Q_{inside}$$

But the enclosed charge is expressed as the volume integral of the charge density ρ

$$Q_{inside} = \int_{box} \rho d\tau$$

So we have,

$$\int_{box} \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_o}\int_{box} \rho d\tau$$

Applying the Divergence theorem to the left-hand side of this equation tells us that

$$\int_{inside} \vec{\nabla}\cdot\vec{E}\, d\tau = \frac{1}{\varepsilon_o}\int_{box} \rho d\tau$$

For any closed box, this means that the integration themselves must be equal, i.e.

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_o}$$

This is the differential form of Gauss's law.

Q. यदि विद्युत क्षेत्र $\vec{E} = 8\hat{x} + 4\hat{y} + 3\hat{z}$ द्वारा प्रदर्शित हो, तो x – y समतल में 200 इकाई क्षेत्रफल के तल द्वारा विद्युत प्रवाह की गणना कीजिए।

If the electric field is given by $\vec{E} = 8\hat{x} + 4\hat{y} + 3\hat{z}$, calculate the electric flux through surface of area 200 units lying in the x-y plane

UPPCS (Mains) 2013 Paper-II

Ans. The electric field vector is given as. $\vec{E} = 8\hat{x} + 4\hat{y} + 3\hat{z}$

The surface area is A = 200 units

The surface lies in the x-y plane.

Determine the area vector

A surface in the x-y plane has its beta vector directed along the z-axis.

Therefore, the area vector $\vec{A}$ is: $\vec{A} = A\hat{z} = 200\hat{z}$

Calculate the electric flux –

The electric flux (ϕ_E) through a surface is given by the dot product of the electric field vector and the area vector:

$$\phi_E = \vec{E}.\vec{A}$$

The vectors into the equation:

$$\phi_E = (8\hat{x} + 4\hat{y} + 3\hat{z}).(200\hat{z})$$

Using the properties of the dot product for unit vectors $(\hat{x}.\hat{z} = 0, \hat{y}.\hat{z} = 0, \hat{z}.\hat{z} = 1)$

$$\phi_E = (8\times 0) + (4\times 0) + (3\times 200)$$

$$\phi_E = 0 + 0 + 600$$

$$\phi_E = 600 \text{ unit}$$

Q. $E = -\nabla V$ सम्बन्ध को स्थापित कीजिए।

Establish the relation $E = -\nabla V$.

UPPCS (Mains) 2012 Paper-II

Ans. Relation between Electric Field and Potential:-

Electric field (E) = $\frac{F}{q}$

The work done (dw) in a moving charge (q) through a distance (dl) in an electric field (E) is

$$dw = F.\, dl = qE\, .\, dl$$

The potential difference (dV) is defined as the work done per unit charge.

$$dV = \frac{dw}{q} = E.dl$$

Gradient of potential:-

For a conservative electric field the potential (V) is scalar function of position. The gradient of potential (∇V) is

$$dV = \nabla V.\, dl$$

The work done by an external agent to move the charge $F_{ext}.\, dl$ and $F_{ext} = -F_{electric}$

$dw_{ext} = -F_{electric}.\, dl = -qE.dl$

Thus, the potential difference is:

$$dV = \frac{dw_{ext}}{q} = -E.dl$$

Comparing this with $dV = \Delta V.\, dl$

$$E = -\nabla V$$

The relation $E = -\nabla V$ shown that the electric field is negative gradient of electric potential.

Q. केन्द्रीय बल क्या है ? दर्शाइये कि केन्द्रीय बल सदैव रूढ़िवादी होते हैं । यदि एक कण केन्द्रीय बल F के अन्तर्गत व्यवहार करता है, तो दर्शाइये कि बल F स्थितिज ऊर्जा के ऋणात्मक प्रवणता के बराबर है तथा इसका कर्ल हमेंशा शून्य होता है।

What are central forces? Show that the central forces are always conservative. If a particle is under the influence of a central force F, show that the force F is the negative gradient of potential energy and curl of it is always zero.

UPPCS (Mains) 2021 Paper-I

Ans.: Central force:- A central force is defined as a force that always act along the line joining a particle and a fixed point, which is called the centre of force. The magnitude of such a force depends only on the direction. Thus, if a particle is at a distance 'r' from the centre the force acting on it is of the form.

$F = F(r)\ \hat{r}$

Central forces are conservative:- A force is said to be conservative if the work done by it in moving a particle from one point to another is independent of the path taken and depends only on the initial and final positions.

For a central force, this condition is naturally satisfied because its magnitude depends only on the radial distance and its direction is always radial.
To prove this mathematically, consider that the work done in moving a particle under a central force from r_1 to r_2 is-

$$W = \int_{r_1}^{r_2} F(r)dr$$

Existence of potential energy for a central force:-
Because the force is conservative, it is possible to define a potential energy function associated with it. Let the potential energy be a function of 'r' only, denoted by V(r). We define the potential energy as-

$$V(r) = -\int_0^r F(r')\,dr'$$

$$\frac{dV}{dr} = -F(r)$$

In vector form, the gradient of V is

$$\nabla V = \frac{dV}{dr}\hat{r}$$

$$-\nabla V = -F(r)\hat{r} = \vec{F}$$

Curl of central force:- Another important criterion for a conservative force is that its curl must vanish. Since a central force is expressible as the negative gradient of a scales potential V, we use the identify

$\nabla \times (\nabla V) = 0$

$\nabla \times \vec{F} = \nabla(-\nabla V) = 0$

This means that the curl of a central force is zero everywhere (except possibly at the origin if the force becomes infinite there). The vanishing curl is thus another proof that central forces are conservative.

Conclusion:- A central force is radial force whose magnitude depends only on the distance from a fixed centre. Such forces are always conservative, because the work done by them is independent of the path of motion. They can be derived from a scaler potential V(r), and mathematically,
$F = -\nabla V$.

Q. स्थिर-वैद्युत क्षेत्र के लिए डाइवर्जेन्स (अपसरण) और कर्ल के व्यंजक लिखिए। इनसे प्वासों और लाप्लास समीकरणों को प्राप्त कीजिए।

त्रिज्याओं r_1 और $r_2 (r_1 < r_2)$ की दो संकेन्द्री सुचालक गोलीय कोशों को क्रमशः V_1 और V_2 विभव पर आवोशित किया जाता है। उन दोनों के बीच अंतराल में विद्युत विभव और अतः विद्युत-क्षेत्र की गणना कीजिए। आन्तरिक कोश पर आवेश की मात्रा को भी ज्ञात कीजिए।

Write expression for divergence and curl of an electrostatic field. From these, obtain Poisson and Laplace equations.

Two concentric conducting spherical shells having radii r_1 and r_2 $(r_1 < r_2)$ are charged to potentials V_1 and V_2, respectively. What is the electrode potential and hence electric field in the space between the shells? Also find the charge on the inner shell.

UPPCS (Mains) 2019 Paper-II

Ans. Divergence and curl of an electrostatic field :-
For electrostatics
Maxwell's equation given

$$\nabla.E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times E = 0$$

Because $\nabla \times E = 0$ there exists a scalar potential V(r) with
$E = -\nabla V$

Poisson and Laplace equation :-
We know the divergence of $E = -\nabla V$
$\nabla E = -\nabla.(\nabla V) = -\nabla^2 V$

$$\nabla^2 V(r) = -\frac{\rho(r)}{\varepsilon_0}$$

$\nabla^2 V(r) = 0$

Two concentric conducting spherical shell:-
Assume spherical symmetry and region between shell $r_1 < r < r_2$ is charge free, so Laplace's equation in spherical symmetry lead to the general radial solution

$$V(r) = A + \frac{B}{r}, \quad r_1 < r < r_2$$

$$V(r_1) = v_1, \qquad V(r_2) = v_2$$

$$B = \frac{(V_1 - V_2)r_1 r_2}{r_2 - r_1}, A = \frac{V_2 r_2 - V_1 r_1}{r_2 - r_1}$$

$$V(r) = \frac{V_2 r_2 - V_1 r_1}{r_2 - r_1} + \frac{(V_1 - V_2)r_1 r_2}{(r_2 - r_1)r}, \; r_1 < r < r_2$$

Electric field between the shells :-

$$E(r) = -\frac{dV}{dr}\hat{r} = \frac{B}{r^2}\hat{r}$$

$$\boxed{E(r) = \frac{(V_1 - V_2)r_1 r_2}{(r_2 - r_1)r^2}\hat{r}} \quad r_1 < r < r_2$$

Charge on the inner shell :-
Use Gauss's law just out side radius r_1 the total charge on the inner shell is

$$Q_1 = \varepsilon_0 \oiint_{r=r_1} E.dA = \varepsilon_0 E(r_1)4\pi r_1^2 = 4\pi\varepsilon_0\beta$$

$$Q_1 = 4\pi\varepsilon_0 \frac{(V_1 - V_2)r_1 r_2}{r_2 - r_1}$$

Q. कूलॉम्ब के नियम को सतत् आवेश वितरण के लिये सदिश प्रारूप में लिखिए। r त्रिज्या के एक गोलीय लूप की परिधि पर q आवेश समान रूप से वितरित है। लूप के लम्बवत् केन्द्र से z दूरी पर विद्युत क्षेत्र ज्ञात कीजिए।

State Coulomb's law in vector form for a continuous charge distribution Charge q is uniformly distributed along the circumference of a circular loop of radius r. Find the electric field at a distance z above the centre along the normal to the loop.

UPPCS (Mains) 2003 Paper-II

Ans. Coulomb's law in vector form for a continuous charge distribution:–
F_{21} is the force exerted on the charge q, them according to coulomb's law.

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} \qquad \text{........ (i)}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{21}^2}\hat{r}_{21} \qquad \text{........ (ii)}$$

$$\vec{r}_{21} = -\vec{r}_{12} \qquad \text{........ (iii)}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\left(-\hat{r}_{12}\right)$$

Put (iii) in (ii),

$$\vec{F}_{12} = -\vec{F}_{21}$$

For a continuous charge distribution the electric field at point π is

$$E(r) = \frac{1}{4\pi\varepsilon_0}\int\frac{r-r'}{|r-r'|^3}\ dq'$$

Where r' run over the source charge position and **dq'** is the element of charge.
Consider a uniformly distributed circular ring of total change '**Q'** along circle with radius 'a'.
To find **E** at the point on the axis at distance **z** above the ring centre.
Choose coordinate so the ring lies in the xy plane centered at the origin.

$$r' = a\cos\phi\,\hat{x} + a\sin\phi\,\hat{y}, dq' = \lambda\, a\, d\phi$$

With linear charge density $\lambda = Q / (2\pi\ a)$
The field point $r = 0\hat{x} + 0\hat{y} + z\hat{z}$
The vector from source to field point is
$r - r' = -a\cos\phi\hat{x} - a\sin\phi\,\hat{y} + z\hat{z}$ and its magnitude is $\sqrt{a^2 + z^2}$ (independent of ϕ)
By symmetry the x, y components cancel upon integrating over ϕ,
So,

$$E_z = \frac{1}{4\pi\varepsilon_0}\int\frac{z}{\left(a^2+z^2\right)^{3/2}}dq'$$

$$= \frac{1\cdot z}{4\pi\varepsilon_0\left(a^2+z^2\right)^{3/2}}$$

The integral of dq' over the ring is Q

$$E(0, 0.z) = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{\left(a^2+z^2\right)^{\frac{3}{2}}}\hat{z}.$$

Q. विद्युत क्षेत्र सदिश $\vec{E}$ का अपसरण और कुंतल ज्ञात कीजिए।
Find the curl and divergence of the electric field vector $\vec{E}$.

UPPCS (Mains) 2003 Paper-II

Ans: For electrostatic charge distribution the electric field is conservative and electric field is a negative gradient of the scalar potential.

$$E(r) = -\nabla v(r), v(r) = \frac{1}{4\pi\varepsilon_0}\frac{zq}{\left(a^2+z^z\right)^{3/2}}$$

Then ∇ × E = 0
Everywhere the field is well behaved (away from the source location).
The curl divergence of a static electric field vector E.
Curl : ∇ × E = 0
Divergence,

$$\nabla\cdot E = \frac{\delta}{\varepsilon_0}$$

Where, δ is the volume charge density.

(c). सिद्ध कीजिए कि एक संरक्षी बल F के लिए $\underline{\nabla}\times\underline{F} = 0$
Prove that for a conservative force F. $\underline{\nabla}\times\underline{F} = 0$

UPPCS (Mains) 2010 Paper-I

Ans.: Let's prove that for a conservative force F, the curl of F is zero, i.e.

Δ × F = 0

A vector field F is conservative if there exists a scalar potential function v(r) such that,

F = –∇v

This means each component of F can be written as

$$F = \left(F_x, F_y, F_z\right) = \left(-\frac{\partial v}{\partial x}, -\frac{\partial v}{\partial y}, -\frac{\partial v}{\partial z}\right)$$

Compute the curl of F
Now compute,

∇ × F = ∇ × (-∇v)

This means the curl of a gradient is always zero, provided the field is continuously differentiable

∇ × F = – ∇ × (∇v) = – 0= 0

Therefore, for any conservative force field $F = \underline{\nabla}\times\underline{F} = 0$

Kinematics, Laws of motion, force and acceleration equations of motion, kinetic and potential energy, linear and angular momentum, work, energy and power, conservation of energy and momentum, Conservative and non-conservative forces

Q. Show that the force field $\vec{F} = \left(2xy + z^2\right)\hat{i} + x^2\,\hat{j} + 2xz\,\hat{k}$ is a conservative force field.
दर्शाइए कि बल क्षेत्र $\vec{F} = \left(2xy + z^2\right)\hat{i} + x^2\,\hat{j} + 2xz\,\hat{k}$ संरक्षी बल क्षेत्र है।

UPPCS (Mains) 2017 Paper-I

Ans. Here, the given force field is F

$$\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$$

Where, $P = 2xy + z^2$
$Q = x^2$
$R = 2xz$

A force field is conservative if its curl is zero, which is equivalent to checking if the following partial derivatives are equal.

$$\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}$$
$$\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}$$
$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$
Now, $\frac{\partial Q}{\partial z} = \frac{\partial}{\partial z}\left(x^2\right) = 0$
$$\frac{\partial R}{\partial y} = \frac{\partial}{\partial y}\left(2xz\right) = 0$$
$$\frac{\partial P}{\partial z} = \frac{\partial}{\partial z}\left(2xy + z^2\right) = 2z$$
$$\frac{\partial R}{\partial x} = \frac{\partial}{\partial x}\left(2xz\right) = 2z$$
$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}\left(2xy + z^2\right) = 2x$$
$$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}\left(x^2\right) = 2x$$
After comparing
$$\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y} \Rightarrow 0 = 0$$
$$\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} \Rightarrow 2z = 2z$$
$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \Rightarrow 2x = 2x$$

(c). Combine components and conclude:
Substituting these partial derivates back into the formula:
$\partial \times F = I\,(0-0) - J\,(2z-2z) + K\,(2x-2x)$
This simplipies to:
$$\partial \times F = 0_I + 0J + 0k = 0$$
Since, the curl to the force field is the zero vector everywhere, the force field F is a cosnservative force field.

Q. What is mean by rotating frame of reference? If frame S' is rotating with a uniform angular velocity with respect to a frame S and the axis of both frames are coincident at t=0, then establish the Galilean transformations between the position co-ordinates, velocity and acceleration.
घूमते हुए निर्देश फ्रेम का क्या आशय है ? यदि फ्रेम S' फ्रेम S के सापेक्ष समान कोणीय वेग से घूर्णन कर रहा है एवं दोनों फ्रेमों के अक्ष t = 0 समय पर सम्पाती है, तो गैलीलीयन रूपान्तरण, स्थिति निर्देशांक, वेग एवं त्वरण के बीच स्थापित कीजिए।

UPPCS (Mains) 2017 Paper-I

Ans. A roatating frame of reference non-inertial is a co-ordinate system used by an observer to measure the position, velocity and acceleration of objects.
A rotating frame of reference-
A rotating frame of reference is a non-inertial frame of reference that is rotating with respect to an inertial frame of reference.
1. Inertial frame- A frame that is either at rest or moving with a constant. Linear velocity Newton's first law of motion holds true in this frame.
2. Non inertial frame- A frame that is a accelerating with respect to an inertial frame. Since rotation involves a charge in the direction of velocity, a rotating frame is always a non-inertial frame. In this frame, fictitious force appear necessary to explain the motion of objects.
Galilean Transformations Between the frames:- Consider an inertial frame S and a frame S rotating with a constant uniform angular velocity ω relative to s. The origin of both frames coincide and their axis were aligned at time t = 0. The following transformations relate the physical quantities measured those S:
Position Co-ordinates:- The position vector r of a particle is the same regardless of which frame it is measured in because the origins of the two frames are coincident:
$$r' = r \rightarrow \text{Position}$$
This is the position Galilean transformation
Velocity:- The velocity measured in frame are reloaded by the following equation;
$$v = v' + \omega \times r'$$
This equation shows that the velocity observed in the inertial frames (s) is the sum of the velocity observed in the rotating frame (s) and the velocity due to the rotation of the frame itself ($\omega \times r'$)
Acceleration- The acceleration measured in frame's 'a' are related by a more complex transformation:
$a = a' + 2\omega \times v' + \omega \times (\omega \times r')$
The terms added to a' on the right-hand side are due to the rotation;
- $2\omega \times v'$: This is the coriolis acceleration term.
- $\omega \times (\omega \times r')$: This is the centripetal acceleration term, directed toward the axis of rotation from the perspective of an observer in the rotating frame's the equation can be rearranged to express a':

$$a' = a - 2\omega \times v' - \omega'$$
The terms subtracted from a associated with the fictitious process experienced in the rotating frame the coriolios force and the centrifugal force.

Q. What is linear momentum ? Prove that in absence of external force on a system of particles, the total linear momentum of the system is conserved.
रेखीय संवेग क्या है? सिद्ध कीजिए कि कणों के निकाय का बाह्य बल की अनुपस्थिति में निकाय का सम्पूर्ण रेखीय संवेग संरक्षित होता है।

UPPCS (Mains) 2017 Paper-I

Ans. Linear momentum- linear momentum is the product of an object is mass and velocity. It’s measure of an object’s tendency to keep moving in a straight line. Linear momentum is represented as
$$p = m \times v$$
Where,
m = mass
v = velocity
Conservation of Linear Momentum:- The law of conservation of linear momentum states that If no

external force acts on a system of particles, the total linear momentum of the system remains constant.
Consider a system of particle's with masses M_1, M_2 ----- M_n and velocities V_1, V_2 ----- V_n. The total linear momentum of the system is:
$p = M_1V_1 + M_2V_2 + \text{--------} + M_nV_n$
According to Newton's second law, the force acting on the i^{th} particle is:
$F_i = M_i \times a_i = M_i \times (dp_i/dt)$
Summing over all particles, the force acting on the whole system is

$$F_{net} = \sum_{i=1}^{n} F_i = \sum_{i=1}^{n} \frac{dp_i}{dt} = \frac{d}{dt}\left(\sum_{i=1}^{n} P_i\right) = \frac{dP}{dt}$$

If no external force acts on the system $F_{net} = 0$
Therefore: $dp/dt = 0$
This implies that the total linear momentum p is constant.
p = constant
Thus, in the obsence of external force, the total linear momentum of the system remains conserved.

Q. एक रॉकेट के सिद्धांत का वर्णन कीजिए। रॉकेट के अंतिम वेग के निम्नलिखित व्यंजक को निर्गमित कीजिए।

$$V = V_0 + v\log_e \frac{M_o}{M}$$

यह भी सिद्ध कीजिए कि जब $\frac{M_o}{M} = c^2$ होगा तब रॉकेट की चाल, इकजस्ट (Exhaust) के चाल की दो गुनी हो जायेगी। (जहाँ संकेतकों के सामान्य अर्थ हैं)
Describe the principle of a rocket. Establish the following relation for the final velocity of a rocket $V = V_0 + v\log_e \frac{M_o}{M}$ also show that the rocket speed is twice the exhaust speed, when $\frac{M_o}{M} = e^2$ (Where the symbols have their usual meanings)

UPPCS (Mains) 2015 Paper-I

Ans. A rocket moves forward by the **Principle of conservation of momentum.**
When the rocket throws out part of its mass (gases) with high speed in the backward direction, an equal and opposite momentum is gained by the rocket in the forward direction. Since no external force acts on the system in free space, the total momentum of the system remains constant and the rocket accelerates as its mass decreases.
Let,
Mass of rocket = M
Velocity of rocket = V
Exhaust speed relative to rocket = v (directed backward)
Initial mass = M_0, final mass = M
Initial velocity = V_0
Momentum conservation
Initial momentum : $P_i = MV$
Final momentum :
• Momentum of rocket (M + dM) (V + dV)
• Momentum of ejected gas (– dM) (V – v)
(Exhaust gas move backward with speed v relative to rocket)
Momentum conservation :
$MV = (M + dM)(V + dV) + (-dM)(V - v)$
Ignoring the second - order term dM, dV

$$0 = MdV + vdM$$

Thus,

$$dV = -\frac{v}{M}dM$$

Integrate from M_0 to M and V_0 to V

$$\int_{V_0}^{V} dV = -v\int_{M_0}^{M} \frac{dM}{M}$$

$$V - V_0 = v\ln\left(\frac{M_0}{M}\right)$$

Hence rocket velocity equation

$$V = V_0 + v\ln\left(\frac{M_0}{M}\right)$$

Condition for Rocket speed = Twice Exhaust speed
Given,

$$\frac{M_0}{M} = e^2$$

Substituting in rocket equation

$$V - V_0 = v.\ln(e^2) = v\cdot 2 = 2v$$

Thus

$$V - V_0 = 2v$$

This means, if the mass ratio of the rocket is e^2, then its speed gain becomes twice the exhaust speed.

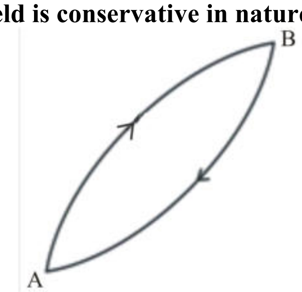
Q. किसी सदिश क्षेत्र के रेखा समाकलन से आप क्या समझते हैं? दिखाइये कि वैद्युत क्षेत्र $\vec{E}$ संरक्षी है।
What do you mean by line integral of a sector field? Show that electric field $\vec{E}$ is conservative.

UPPCS (Mains) 2012 Paper-II

Ans. Line integral of a vector field:- The line integral of a vector field represents the work done by the field in moving a particle along a specific path.
The line integral of a vector field $\vec{F}$ along curve C is

$$\int \vec{F}.\vec{dl}$$

This represent the work done by force $\vec{F}$ in moving a particle.
Electrical field is conservative in nature:-



Consider a charge Q placed in an electric field placed at points A and B. A to B is a closed path work is done by electric field to move a test charge from point A to B.
The line integral of electric field along A to B $\int_A^B \vec{E}.\vec{dl}$

The line integral of electric field along B to A

$$\int_B^A \vec{E}.\vec{dl}$$

Hence, line integral along closed path is the sum of two integrals.

$$\int_A^B \vec{E}.\vec{dl} + \int_B^A \vec{E}.\vec{dl}$$

But,

$$\int_A^B \vec{E}.\vec{dl} + \int_B^A \vec{E}.\vec{dl}$$

This shows that line integral of an electric field along a closed path is zero.

Hence, work done by electric field is independent of path which means it depends only on points A and B.

Hence, we can prove that electric field is conservative in nature.

Q. 1.02 MeV वाले गतिज ऊर्जा के एक इलेक्ट्रॉन की चाल एवं संवेग की गणना कीजिये। इलेक्ट्रॉन का विराम द्रव्यमान 9.1×10^{-31} कि. ग्रा. है।

Calculate the speed and momentum of an electron of kinetic energy 1.02 MeV. Rest mass of electron is 9.1×10^{-31}kg.

UPPCS (Mains) 2018 Paper-I

Ans. Given,

Kinetic energy (K) = 1.02 MeV

$= 1.02\times10^{6}\times1.602\times10^{-19}$

$= 1.634\times10^{-13}$ J

Rest mass electron (m_0) = 9.1×10^{-31} kg

Speed of light (C) = 3×10^{8} m/sec

Speed (v) = ?

Momentum (p) = ?

$$E = E_0 + K.$$

$$E = m_0C^2 + K$$

Where,

$$E = \text{Total energy}$$

$$E_0 = \text{Rest mass energy}$$

$$K = \text{Kinetic energy}$$

$$E_0 = m_0C^2$$

$$= 9.1\times10^{-31}\times\left(3\times10^{8}\right)^2$$

$$E_0 = 8.19\times10^{-14}\,J$$

$$E = 9.1\times10^{-31}\times\left(3\times10^{8}\right)^2 + 1.634\times10^{-13}$$

$$= 9.1\times10^{-31}\times9\times10^{16} + 1.634\times10^{-13}$$

$$= 8.19\times10^{-14} + 1.634\times10^{-13}$$

$$E = 2.453\times10^{-13}\,J$$

$$\gamma = \frac{E}{E_0}$$

$$\gamma = \frac{2.453\times10^{-13}}{8.19\times10^{-14}}$$

$$\gamma = 2.995$$

$$v = C\sqrt{1-\frac{1}{\gamma^2}}$$

$$v = 3\times10^{8}\sqrt{1-\frac{1}{(2.995)^2}}$$

$$v \approx 2.83\times10^{8}\,m/sec$$

Momentum (p) = $\gamma m_0 v$

$$= 2.995\times9.1\times10^{-31}\times2.83\times10^{8}$$

$$p = 7.713\times10^{-22}\,kg.m/sec$$

Q. एक ठोस बेलन व एक ठोस गोले की त्रिज्याओं का अनुपात 1 : 2 है। यदि उनके स्वयं की अक्ष के परितः जड़त्व आघूर्ण समान हो तो उनके द्रव्यमानों का अनुपात क्या होगा?

The ratio of radii of a solid cylinder and a solid sphere is 1 : 2 if the moment of inertia about their own axis is equal then what will be the ratio of their masses?

UPPCS (Mains) 2018 Paper-I

Ans. The Moment of Inertia formulas Solid cylinder about its own axis

$$I_{cyl} = \frac{1}{2} m_{cyl} R^2_{cyl}$$

Solid sphere about its diameter

$$I_{sph} = \frac{2}{5} M_{sph} R^2_{sph}$$

Given Ratio and Equality

Radii ratio

$$R_{cyl} \sim R_{sph} = 1:2 \Rightarrow R_{sph} = 2R_{cyl}$$

Moments of inertia are equal

$$I_{cyl} = I_{sph}$$

Set up the Equation

Substitute the formulas and the radius relation

$$\frac{1}{2} M_{cyl} R^2_{cyl} = \frac{2}{5} M_{sph} \left(2R_{cyl}\right)^2$$

Simplify the right – hand Side

$$\frac{2}{5} M_{sph} \cdot 4R^2_{cyl} = \frac{8}{5} M_{sph} R^2_{cyl}$$

So the equation become

$$\frac{1}{2} M_{cyl} R^2_{cyl} = \frac{8}{5} M_{sph} R^2_{cyl}$$

Cancel R^2_{cyl} from both sides.

$$\frac{1}{2} M_{cyl} = \frac{8}{5} M_{sph}$$

Soles for the Mass Ratio

Multiply both sides by 2.

$$M_{cyl} = \frac{16}{5} M_{sph}$$

Thus $M_{cyl} : M_{sph} = 16 : 5$

Q. प्रत्यास्थ तथा अप्रत्यास्थ संघर्ष से आप क्या समझते हैं ? m_1 और m_2 द्रव्यमान के दो पिण्ड क्रमशः u_1 व u_2 वेग से गतिशील है। प्रत्यास्थ संघर्ष के पश्चात् उनके वेग ज्ञात कीजिये।

What do you understand by the elastic and inelastic collisions? Two bodies of masses m_1 and m_2 are moving with velocities u_1 and u_2 respectively. Find their velocities after the elastic collision.

UPPCS (Mains) 2018 Paper-I

Ans. Elastic and Inelastic collisions

Elastic collision:

- **Momentum is conserved :** The total momentum before the collision equals the total momentum after the collision.
- **The Kinetic Energy is conserved :** The total Kinetic energy before the collision equals the total Kinetic energy after the collision.
- **Real-world examples :** Collision between atomic particles, billiard balls (approximately), or steel ball bearings.
  There is no permanent deformation or generation of heat.

Inelastic Collision :

- **Momentum is conserved :** The total momentum before the collision equals the total momentum after the collision.
- **Kinetic Energy is NOT Conserved :** Some Kinetic energy is transformed into other forms of energy, such as heat sound, or energy of deformation.
- **Perfectly Inelastic Collision :** This is a specific case where the two objects stick together after the collision and move with a common velocity. This results in the maximum possible loss of Kinetic energy.

Velocities after elastic friction

Suppose two bodies which mass is m_1 and m_2 and initial velocity is u_1 and u_2, final velocity is v_1 and v_2.

The Principle of conservation of momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

Principle of conservation of Kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Substituting these equation, we get the formula.

$$\boxed{v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2}$$

$$\boxed{v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2}$$

1. If m_1 is equal to m_2

$v_1 = u_2$, $v_2 = u_1$

That is, when the Masses are equal, the bodies change their velocities after collision.

2. If m_2 is very large and $u_2 = 0$ (like hitting walls)

v_1 □ u_1, v_2 □ 0

That is, the light body returns with the same speed.

3. If $m_1 >> m_2$ and $u_2 = 0$

v_1 □ u_1, $v_2 = 2u_1$

That is, the velocity of heavier bodies, remaining unaffected and light bodies have velocity double.

Q. Show that the angular momentum of a system acted upon by a central force is conserved. Hence prove that the real velocity of a planet moving around the sun is constant.

दर्शाइये कि केन्द्रीय बल द्वारा कार्यरत निकाय का कोणीय संवेग संरक्षित होता है। तब सिद्ध कीजिये कि सूर्य के परितः गतिशील ग्रह का क्षेत्रीय वेग नियत रहता है।

UPPCS (Mains) 2016 Paper-I

Ans. Angular momentum for a central force :

$$\vec{F} = F(r)\hat{r}$$

Then, torque on this planet

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = \vec{r} \times F(r)\hat{r} \qquad \because \text{ from equation ....(i)}$

$\vec{\tau} = \vec{r} \times F(r)\frac{\vec{r}}{r} \qquad \because \hat{r} = \frac{\vec{r}}{r}$

$\vec{\tau} = \frac{F(r)}{r}(\vec{r} \times \vec{r})$

$\vec{\tau} = 0 \qquad \because \vec{r} \times \vec{r} = 0$

$\frac{d\vec{L}}{dt} = 0, \ \vec{L} = \text{constant}$

Hence, under central force angular momentum of any planet or particle remains constant.

Areal velocity of a planet is constant

Let the magnitude of angular momentum

$L = mr^2\theta$

The areal velocity

$\frac{dA}{dt} = \frac{1}{2}r^2\theta$

Substitute $r^2\theta = \frac{L}{M}$

$\frac{dA}{dt} = \frac{L}{2M} = \text{constant}$

So,

$\frac{dA}{dt} = \text{constant}$

Q. राकेट का कार्य सिद्धान्त क्या है? राकेट के अंतिम वेग के लिए निम्न सूत्र की व्युत्पत्ति कीजिए।

$$\mathbf{V = V_0 + v \log_e \frac{M_0}{M}}$$

जहाँ प्रयुक्त पदों के सामान्य अर्थ है। राकेट के वेग को कितना बढ़ाया जाये कि वह पृथ्वी के आकर्षण क्षेत्र से पलायन कर जाये? द्वि–स्तरीय राकेट का एक स्तरीय राकेट की तुलना में क्या लाभ है?

What is the working principle of a Rocket? Derive the following formula for the final velocity of a rocket.

$$\mathbf{V = V_0 + v \log_e \frac{M_0}{M}}$$

Where the terms have their usual meaning. How much should the velocity of the rocket be increased so as to escape from the earth's attracting field? What is the advantage of two stage rocket in comparison to single rocket?

UPPCS (Mains) 2021 Paper-I

Ans. Working principle of a rocket:-

A rocket works on the basic principle of Newton's third law of motion, which states that for every action, there is an equal and opposite reaction. In a rocket, the burning of fuel produce hot gases at high pressure. These gases escape rapidly through the nozzle in the backward direction. The backward ejection of gases constitutes the action, and the rocket experiences a forward reaction, which propels it upward.

Unlike an aircraft, a rocket does not need air to push against, it carries both fuel and oxidizer. Therefore, it can work even the vacuum of space. The motion of the rocket arises solely from the conservation of linear momentum, as the momentum lost by the ejected gases equals the momentum gained by the rocket.

Derivation of the rocket equation:-

Consider a rocket of mass M moving with velocity V. During a small interval of time, it ejects a small mass dM of burnt gases with an exhaust velocity V relative to the rocket. Because the mass of the rocket decreases, we take dM as negative.

Before ejection, the momentum of the system is –

$$P_{initial} = MV$$

After ejection, the rocket's mass becomes M + dM and its velocity becomes V + dV. The ejected gas of mass-dM moves backward with velocity V – v (because v is relative to the rocket).

So,

$MV = (M - dm)(V + dV) + dm(V - v)$

By the law of conservation of momentum.

$P_{initial} = P_{final}$

Expanding and neglecting second-order small quantities.

$MdV = vdM$

Rewriting,

$$dV = \frac{vdM}{M}$$

Integrating between the limits

When mass is M_0, velocity is V_0

When mass reduces to M, velocity becomes;

$$\int_{V_0}^{V} dV = v\int_{M_0}^{M} \frac{dM}{M}$$

Thus,

$$V - V_0 = v\,ln\left(\frac{M_0}{M}\right)$$

Since $ln(M/M_0) = -ln(M_0/M)$, we write the final result as:

$V = V_0 + v\,ln(M_0/M)$

The equation is known as the Tsiolkovsky Rocket equation.

Here,

V = Final velocity

V_0 = Initial velocity

v = Exhaust velocity of gases relative to the rocket

M_0 = Initial total mass of the rocket

M = final mass after fuel is consumed

Initial in velocity required for a rocket to escape earth

Escape velocity is defined as the minimum speed that a body must have in order to overcome the gravitational attraction of the earth and reach infinite distance without further propulsion.

It is given by:-

$$v_{esc} = \sqrt{2gR}$$

Using $g = 9.8\ m/s^2$

$R = 6.37 \times 10^6$ m

$v_{esc} \approx 11.2$ km/s.

Therefore, a rocket must be increased in velocity by about 11.2 km/s (neglecting air resistance and gravity losses) in order to escape from the earth's gravitational field.

Advantages of a two-stage rocket over a single-stage rocket:-

For two - stage rocket several important advantages:-

1. Thrust and final velocity:-

According to the rocket equation, the velocity gained depends on the ratio M_0/M.

In a two stage rocket, after the fuel of the first stage is consumed, the empty structure is discarded. This improves the mass ratio for the second stage, enabling much higher speeds than a single-stage rocket.

2. Reduced dead weight:-

In a single-stage rocket, even after fuel is burnt, the heavy empty tanks and engines remain attached to the rocket and become dead weight.

In a multi-stage rocket, this useless mass is shed gradually greatly improving efficiency.

3. Better fuel efficiency:-

Each stage can be designed with engines optimized for its operating environment:

- The first for high thrust in dense atmosphere
- The second stage for high efficiency in vaccum.

4. Economical and practical:

Achieving escape velocity with a single-stage rocket would require a huge mass of fuel and a massive structure, making it having impractical.

Staging makes space missions technically feasible and economically viable.

Q. चलित द्रव की विभिन्न ऊर्जा रूप कौन-कौन से होते हैं? इन ऊर्जाओं के संबंधों को प्रति इकाई द्रव्यमान के रूप में लिखिए। यह भी दर्शाइए कि इन ऊर्जाओं का प्रति इकाई द्रव्यमान कुल योग स्थिर होता है।

What are the various forms of energies of a liquid in motion? Write their relations in terms of unit mass. Also show that sum of these energies per unit mass is constant.

[Ashram Paddhati (Mains)-2021]

Ans. : Energies of a liquid in motion (Bernoulli's principle):

It is pertains to Bernoulli's principle, which is an application of the Law of Conservation of Energy for a flowing fluid.

1. The various forms of energy in a liquid in motion:

A liquid (or any fluid) flowing under conditions of steady, incompressible, and non-viscous flow possesses three primary forms of mechanical energy. The sum of these three forms represents the total mechanical energy of the fluid element.

(i). Pressure Energy (E_{Pr})

This is the energy stored in the liquid due to the work done by the surrounding pressure (P) on the fluid element.

It is often called flow work or flow energy because it is required to push the fluid cross a boundary.

Source: The static pressure exerted by the surrounding fluid on the fluid element.

(ii). Kinetic Energy (E_K) :
This is the energy possessed by the liquid due to its mass (m) and velocity (v). It represents the energy of motion.
Source: The movement of liquid itself.
(iii). Potential Energy (E_P) :
This is the energy stored in the liquid due to its position (z) or elevation relative to defined reference level (datum).
Source: The gravitational force acting on the liquid mass.
2. Relation in terms of unit mass (Specific Energy):
In fluid dynamic, it most useful to express these energies per unit mass (E/m) as they represent intrinsic properties of the flow independent of the volume or mass considered. These are also known as specific Energy (units of J/kg or m^2/s^2).
(i). Specific Potential Energy (E_P/m):
The formula for potential energy is

$$E_P = m.g.z$$

$$\frac{E_P}{m} = \frac{m.g.z}{m} = g.z$$

(ii). Specific Kinetic Energy (E_K/m):
The formula for kinetic energy is $E_K = \frac{1}{2}m.v^2$

$$\frac{E_K}{m} = \frac{\frac{1}{2}m.v^2}{m} = \frac{v^2}{2}$$

(iii). Specific Pressure Energy (E_{Pr}/m):
The formula for pressure energy (Flow work) is E_{pr} = P.V, where V is the volume. Since volume V is related to mass m and density ρ by

$$V = \frac{m}{\rho}$$

$$\frac{E_{Pr}}{m} = \frac{P.V}{m} = \frac{P.(m/\rho)}{m} = \frac{P}{\rho}$$

3. Proof: Sum of these Energies Per Unit Mass is Constant:
The statement that the sum of these energies per unit mass is constant is known as Bernoulli's principle. This principle is derived directly from the application of the Law of Conservation of Energy to fluid flow.
(i). Total Specific Energy:
The Total Energy Per Unit Mass (E_T/m) at any point in the flow is the sum of the specific forms:

$$\frac{E_T}{m} = g.z + \frac{v^2}{2} + \frac{P}{\rho}$$

(ii). The Conservation Principle:
For a flow to which Bernoulli's principle applies (i.e., ideal, non-viscous flow), the energy within the fluid system cannot be created or destroyed. Therefore, the total energy carried by the fluid element must be the same of every point along its streamline. Considering two arbitrary points, point 1 and point 2 along a single streamline:

$$\left(g.z_1 + \frac{v_1^2}{2} + \frac{P_1}{\rho}\right) = \left(g.z_2 + \frac{v_2^2}{2} + \frac{P_2}{\rho}\right)$$

or

$$g.z + \frac{v^2}{2} + \frac{P}{\rho} = \text{constant}$$

(iii). Physical Interpretation:
This constant relationship shows the interchangeability of the three forms of energy.
• If the fluid rises to a higher elevation (increase in z, $E_P\uparrow$), either its velocity must decrease (kinetic energy↓) or its static pressure must decrease (pressure energy↓).
If the fluid flows into a narrower section (increase v, $E_K\uparrow$), its static pressure (P) must be drop to compensate.
This fundamental equation proves that the sum of the potential, kinetic, and pressure energies per unit mass remains constant throughout the flow for an ideal liquid.
Note: The constant sum can also be expressed in terms of energy per unit weight (called total head), obtained by dividing the specific energy equation by g:

$$z + \frac{v^2}{2g} + \frac{P}{\rho g} = \text{Constant Head}$$

Q. एक छल्ला, एक डिस्क, एक ठोस बेलन, एक गोलीय कोश तथा एक ठोस गोला जिनकी त्रिज्यायें तथा द्रव्यमान समान हैं, को एक नत समतल के उच्चतम बिन्दुओं में विरामावस्था से बिना फिसले लुढ़कने के लिए छोड़ दिया जाता है। उपरोक्त पिण्डों के वेग हेतु एक सामान्य व्यंजक ज्ञात कीजिये और नत समतल के निम्न बिन्दु पर पहुँचने का उनका क्रम ज्ञात कीजियें ।
A ring, a disc, a solid cylinder, a spherical shell and solid sphere of the same mass and same radius are allowed to roll without slipping from rest state of an inclined plane. Find the general expression for the velocity of the bodies and also find their order of reaching to the bottom of the inclined plane.
UPPCS (Mains) 2020 Paper-I

Ans. Use energy conservation and no-slip condition.
From top height h.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2,$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2, \text{ with no-slip}, v = \omega R$$

$$v = \sqrt{\frac{2gh}{1+\frac{I}{mR^2}}}$$

For common bodies (same m, R, h)
• Thin ring (hoop) : $I = mR^2$

$$v = \sqrt{gh}$$

• Solid cylinder/disk $I = \frac{1}{2}mR^2$

$$v = \sqrt{\frac{4}{3}gh}$$

• Thin spherical shell : $I = \frac{2}{3}mR^2$

$$v = \sqrt{\frac{6}{5}gh}$$

• Solid sphere : $I = \frac{2}{5}mR^2$

$$v = \sqrt{\frac{10}{7}gh}$$

Larger v → reaches first order (First → last):
Solid sphere > solid disk/cyclinder > thin spherical shell > thin ring (hoop).

Q. A particle of mass M moving with a velocity u collides head on with a stationary particle of mass m. After the collision, they begin to move with the velocities V and ν respectively in the same direction. If the collision is elastic, then prove that,

$$\nu = \frac{2u}{\left(1+\frac{m}{M}\right)}$$

एक M द्रव्यमान का कण u वेग से एक m द्रव्यमान के स्थिर कण से सीधे टकराता है। संघट्ट के पश्चात् वे उसी दिशा में क्रमशः V तथा ν वेग से गतिशील हो जाते हैं। यदि संघट्ट प्रत्यस्थ है, तो सिद्ध कीजिये।

$$\nu = \frac{2u}{\left(1+\frac{m}{M}\right)}$$

UPPCS (Mains) 2016 Paper-I

Ans. The principle of conservation of momentum states that the total momentum before the collision is equal to the total momentum after the collision.
Applying conservation of momentum:

$P_{initial} = P_{final}$

$Mu + m(0) = MV + m\nu$

$Mu = MV + m\nu$ ....(i)

Apply conservation of kinetic energy is also conserved.

$KE_{initial} = KE_{final}$

$$\frac{1}{2}Mu^2 + \frac{1}{2}m(0)^2 = \frac{1}{2}MV^2 + \frac{1}{2}m\nu^2$$

$Mu^2 = MV^2 + m\nu^2$ .... (ii)

Solve for the final velocities rearrange equations (i) and (ii):

From (i) $M(u - V) = m\nu$ ....(iii)

From (ii) $M(u^2 - V^2) = m\nu^2$

$M(u - V)(u + V) = m\nu^2$ .... (iv)

Divide equation (iv) by equation (iii):

$$\frac{M(u-V)(u+V)}{M(u-V)} = \frac{m\nu^2}{m\nu}$$

$V = \nu - u$ ....(v)

Substitute equation (v) into equation (i),

$Mu = M(\nu - u) + m\nu$

$Mu = M\nu - Mu + m\nu$

$2Mu = \nu(M + m)$

$$\nu = \frac{2Mu}{M+m}$$

Simplify the expression,

$$\nu = \frac{\frac{2Mu}{M}}{\frac{M+m}{M}}$$

$$\nu = \frac{2u}{\left(1+\frac{m}{M}\right)}$$

Q. व्यूत्क्रम वर्ग-नियमानुसार प्रतिकर्षी बल के प्रभाव में एक कण की गति का वर्णन कीजिए।

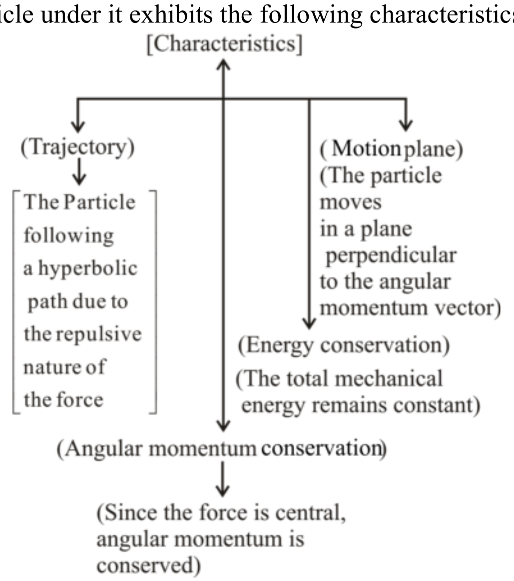
Describe the motion of a particle under repulsive inverse square law force.

UPPCS (Mains) 2008 Paper-I

Ans.: Motion under Repulsive Inverse Square Law force:–
A repulsive inverse square law force is described by

$$\left[F(r) = \frac{k}{r^2}\right]$$

Where,
K > 0 and r is the distance from the force center. This force is central and conservative, and the motion of a particle under it exhibits the following characteristics:-



Q. संरक्षित और असंरक्षित बलों का अन्तर उदाहरण सहित समझाइये।

Differentiate between conservative and non-conservative forces with examples.

UPPCS (Mains) 2008 Paper-I

Ans.: In Newtonian physics, there are generally two types of forces; conservative and non-conservative forces.

A conservative force is a force in which the work done moving a particle from one position to another depends on the initial and final positions and not on the path. If follows the law of conservation of energy. The total work done by a conservative force around a closed path is zero.